OAR:
The Basic Research Agency
of The United States Air Force

VOLUME II NUMBER 18 18 NOVEMBER 1963

THE FOURTH STATE OF MATTER

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The ionized state of a gas turns out to be the most common form of matter in the universe. It is no coincidence, therefore, that a major study of its properties has become one of the most concentrated research efforts under the Joint Services Electronics Program, AFOSR Project 4751.

It was known many years ago that the microwave capacity of ionized gases would create terrific communication problems for a vehicle returning to earth from the outer regions of the atmosphere. The phenomenon was highlighted recently when our astronauts (Sheppard and Company) failed to communicate during those crucial moments of the atmosphere reentry stage. The condition known as "blackout" is actually a plasma resonance and has been the hard-core subject of research at MIT under Professor Sanborn C. Brown of the Research Laboratory of Electronics. Professor Brown is leading a major effort in probing the "fourth state of matter."

The plasma-frequency condition occurs because of the very nature of the plasma itself. Since a plasma is a neutral collection of electrons and ions, the coulomb attraction between the charged species provides a coupling between the electrons and the ions of such a nature that, if an electron is moved from its equilibrium position by the application of an external electric field, neighboring ions will return it to its equilibrium position in exactly the same manner as a weight hung on a spring must be returned by the restoring force of the spring. In the case of such a weight on a spring, the stiffer the spring, the higher the frequency of oscillation; just so with an electron--the higher the density of charged particles, the greater the

restoring force and the higher the frequency of its natural oscillation in the field of the ions. When a radio wave hits a plasma, therefore, it displaces the electrons; and if it hits the natural plasma frequency of the collection of electrons and ions, its energy will all be dissipated in plasma resonance, and none will be left over for transmission through such an ionized gas. At 10-cm wavelength, the critical density for this condition is around 5×10^{10} elec/cc, and the frequency increases with the square root of the charged-particle density. One of the frustrating conditions of physics at the moment is that many of the interesting plasma phenomena occur for densities between 10^{13} and 10^{15} elec/cc; to probe these densities with any kind of electromagnetic radiation requires wavelengths in the region between a millimeter and a micron.

The tremendous amount of effort necessary to produce ultrahigh-frequency generators in the millimeter region leads one to predict even greater problems in trying to go to shorter wavelengths. Rather than trying to overcome the really tremendous technical problem of going to submillimeter radio waves, Professor Brown's group at the Research Laboratory of Electronics, MIT, has concentrated on adapting very long infrared rays to this plasma diagnostic problem.

The development of a technique to use this very far-infrared radiation involved utilizing some of the newest materials and detection methods, although the optical arrangement is based directly on refractometer devices which have been known for the last 50 to 80 years. Choosing an infrared wavelength of 0.3 mm, the MIT group first had to find materials suitable for infrared transmission. At this wavelength, the only materials which are good transmitters are polyethylene, diamond, and crystal quartz. All three of these materials were incorporated into the final device--the diffraction gratings were made of polyethylene; the windows for the plasma tubes utilized crystal quartz; and the detector had a diamond window. The source of radiation was a high-pressure mercury-arc street lamp. The radiation was broken up into its component wavelengths by a blazed diffraction grating which put the energy in the first order of the spectrum. The +1 and -1 first-order spectra passed along different paths so that the plasma to be studied could intercept one of the paths, and the phase difference corresponding to the density of plasma passed through, compared with the path in which no plasma was contained, constituted the measurement. Actual densities up to 5×10^{13} on the top end and overlapping K-band measurements at the low-density region, for calibration purposes, constitute the measurements so far. Improved techniques may well push the density measurements up another factor of 10.

Of particular importance to this device is the development in recent months of Indium Antimonide photoconductors. These detectors, which start to be sensitive around 0.10 mm and have been tested up to several millimeters wavelength, constitute a real breakthrough for these radiation wavelengths. Small crystals of Indium Antimonide must be operated in liquid-helium baths in magnetic fields of a few thousand gauss; thus, not only does the detector require modern cryogenic techniques, but superconducting magnets as well. To date, these diagnostic techniques have been applied only to steady-state plasmas. But it is hoped that future work will lead to devices which will be useful also for pulsed and transient diagnostic studies.

generation of submillimeter electromagnetic radiation

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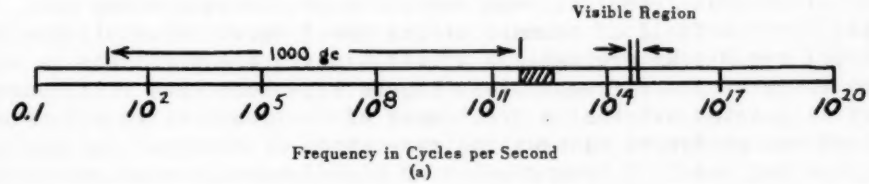
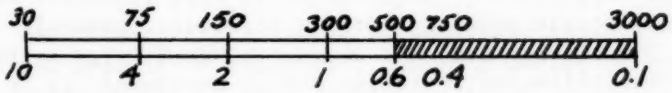
The problem of how to obtain useful amounts of submillimeter electromagnetic radiation continues to be one of the most important and challenging subjects for Air Force supported basic research. The submillimeter region, as discussed here, is considered to be that portion of the electromagnetic frequency spectrum between about 500 and 3000 gc, as shown in Figure 1. (One gc, for "gigacycle," is 10^9 cycles per second; it is also often abbreviated "kmcs" for kilomegacycles per second). This frequency range corresponds to wave lengths between 0.1 and 0.6 millimeters; hence the label of "submillimeter."



Frequency in Cycles per Second
(a)

Frequency (ν) in gc.



Wavelength (λ) in mm.
(b)

$$\lambda_{(mm)} = \frac{300}{\nu_{(gc)}}$$

(c)

Fig. 1. (a) The electromagnetic frequency spectrum with the submillimeter "gap" indicated by the shaded portion. (b) The millimeter and submillimeter region on an expanded scale. (c) Equation for conversion between wavelength in millimeters and frequency in gigacycles.

Immediately below this frequency region (from 0.6 to about 2 millimeters), oscillators have now been developed which produce coherent radiation at power levels of at least several milliwatts. However, these oscillators still require considerable study and design effort

because, until very recently, these wave lengths could only be obtained from overtones of a lower frequency rather than from the fundamental. This results in very inefficient energy conversion for the system.

Oscillators producing fundamental frequencies below 150 gc and having power levels of 10 milliwatts or more are now generally available. While these power levels are quite low, they are, nevertheless, sufficient for many important investigations, such as microwave spectroscopy. Power levels reaching many kilowatts, of course, are available at frequencies lower than these. Above 3000 gc it has long been possible to obtain sufficient energy for many important investigations by filtering the desired frequencies from high-energy incoherent sources (as with arc-source spectrometers). In recent years the development of the MASER has provided coherent radiation at certain discrete frequencies both above and below the submillimeter region. Exploitation of the frequency range between 500 and 3000 gc, however, is still denied us primarily because of experimental difficulties rather than the result of theoretical barriers.

The importance of developing coherent electromagnetic wave generators in the "forbidden" submillimeter region stems not only from the inherent scientific challenge the problem presents, but also from the number of current technical requirements such generators can meet. For example, in the field of communications the frequencies available between 0.1 and 0.6 mm are two and a half times those available in the entire range of lower frequencies (Figure 1). More than this, however, it may be pointed out that a great deal of the progress in understanding the fundamental forces controlling many kinds of chemical and physical processes has resulted from studies of high-frequency electromagnetic resonance phenomena in molecules, atoms and atomic nuclei. A classic example of this occurred in 1947 when W. E. Lamb with R. C. Retherford,[1] using microwave spectroscopic studies of hydrogen, showed that two types of electron configurations previously considered equal in energies were actually different. The difference, though small, is very important in modern quantum-mechanical atomic models. For his discovery, commonly called the "Lamb shift," Professor Lamb was awarded a Nobel Prize in 1955. If suitable sources of submillimeter radiation can be developed, it is likely that other similarly impressive advances in our knowledge will be made.

One very important area currently in need of attention, which can be opened for the first time by submillimeter wave spectroscopy, is a more complete understanding of crystal chemistry, including crystal growth. Another equally important area which can be studied only by the use of submillimeter waves is the properties of very dense plasmas. Here measurements are made of changes in phase angles and directions of energy wave propagation which result from the interaction of electromagnetic waves with the plasma. From these measurements important plasma properties can be deduced. Standard techniques using

wire probes are often unsuitable in dense plasmas because the property to be measured is destroyed by the presence of the probe.

A variety of ways of generating submillimeter radiation have been proposed and a number tried. Some that have achieved limited success and others that hold promise for the future include gas-discharge oscillators, miniaturization of conventional cavity oscillators, backward-wave oscillators, MASERs, parametric amplifiers, nonlinear RLC and ferromagnetic devices, and relativistic electron sources.

Two types of gas-discharge oscillators have been studied extensively. The first represented the earliest technique used to produce submillimeter radiation. It is called a "mass radiator" and consists of a direct-current arc through which tiny balls, with diameters equal to the wave length of the desired radiation, are passed. Wave lengths of 0.6 mm were produced in 1895 and, in 1924, wave lengths as short as 0.13 mm were obtained. The best power produced by this method, however, was only about 30 milliwatts. Besides producing low power, this oscillator is very unstable and, of course, excessively noisy.

A second gas-discharge method was developed by Professor P. D. Coleman, University of Illinois, as a result of research supported in part by the General Physics Division of AFOSR.[2] Professor Coleman has successfully produced radiation at about 1 mm with an average power level of about 0.1 watt. His method involves inducing ionization in a gas by application of a strong nonlinear radio-frequency signal. The submillimeter radiation is produced by harmonics of the radio-frequency signal resulting from its nonlinear characteristics. Tests have shown that the radiation is highly coherent, having a coherent/incoherent ratio of more than 100:1.

The classical way for many years of producing higher-frequency radiation has been scale reduction of conventional cavity oscillators and associated equipment. Under a series of research contracts sponsored by the General Physics Division, AFOSR,[3] Professor Hans Motz, Oxford University, England, has pioneered in the development of ultrashort wave generators in this way. He recently constructed a klystron which operates at 2.5 mm and produces about 40 milliwatts of power. The primary problem with this approach, however, is illustrated by the size of a cylindrical wave guide for this frequency which has a diameter of only about 2 mm. At higher frequencies, the dimensions would be even smaller. In fact, most experts now agree that scale reduction as a way to obtain shorter wave-length oscillations has just about reached the practical limit of the machinist's art.

Another method which has recently achieved considerable success is the backward-wave oscillator (bwo) developed by Dr. A. Karp while at

the Bell Telephone Laboratories.[4] This device (Figure 2) consists of a ridged or ladder-like structure, long in one dimension compared to the other two. The two small dimensions, of course, are comparable to the wave lengths being generated; but the construction is considerably simpler than when all three dimensions must be so small, as is necessary

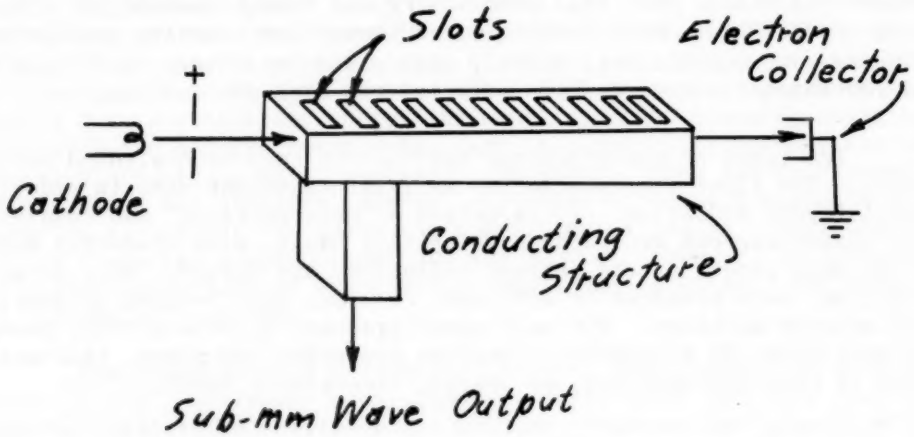


Fig. 2. Simplified Diagram of a Backward-Wave Oscillator.

in simple scale reduction procedures. The high frequencies are generated as a result of the interaction between the electron beam and currents induced in the ridged or ladder-like structure. Backward-wave oscillators are now available on the commercial market from various manufacturers. Sylvania puts out a 110-gc bwo which produces several milliwatts of power. Bendix and Sperry Rand manufacture such oscillators covering the frequency range of 40 to 120 gc. And the CSF Company of France has introduced "O"-type bwo's producing several milliwatts at frequencies of 150, 300, 425, and 500 gc. Recently, it has even come out with one version producing weak but useable power at 750 gc.

The spectacular success in the development of infrared and optical MASERs during the past five years indicates that this approach holds much promise for generating coherent radiation in the submillimeter region. The idea here is to utilize the beat frequency produced by two MASERs oscillating at nearly the same frequency. Professor R. J. Strain, University of Illinois, supported by the General Physics Division, AFOSR,(5) is studying this method by projecting two LASER beams into an anisotropic dielectric so as to produce a difference frequency in the submillimeter region. The current program involves mixing the output of a pink ruby LASER, which can be thermally tuned over a range of 1900 gc, with a discrete frequency emitted from a red ruby LASER. If successful, coherent sources from zero to 6800 gc will be available.

Parametric amplifiers are considered in connection with submillimeter generators because, although they are not oscillators, they offer the promise of raising the power obtainable from low-energy oscillators to useful levels. This type amplifier does not use conventional electronic tubes to provide amplification through valve-like action. Rather, its operation is based upon increasing the amplitude of the input signal by adding energy in phase through an auxiliary circuit, called the "pump" circuit. Figure 3 shows a schematic diagram of a parametric amplifier circuit. The pump adds energy to the input signal by changing the capacitance of the pump circuit in phase with the signal frequency. The "idler" component is added to smooth out the effect of any phase difference between the pump and signal frequencies. The result is that the amplitude of the voltage signal across the input terminals (junctions (1) and (2) in Figure 3) grows and amplification of the input signal is achieved.

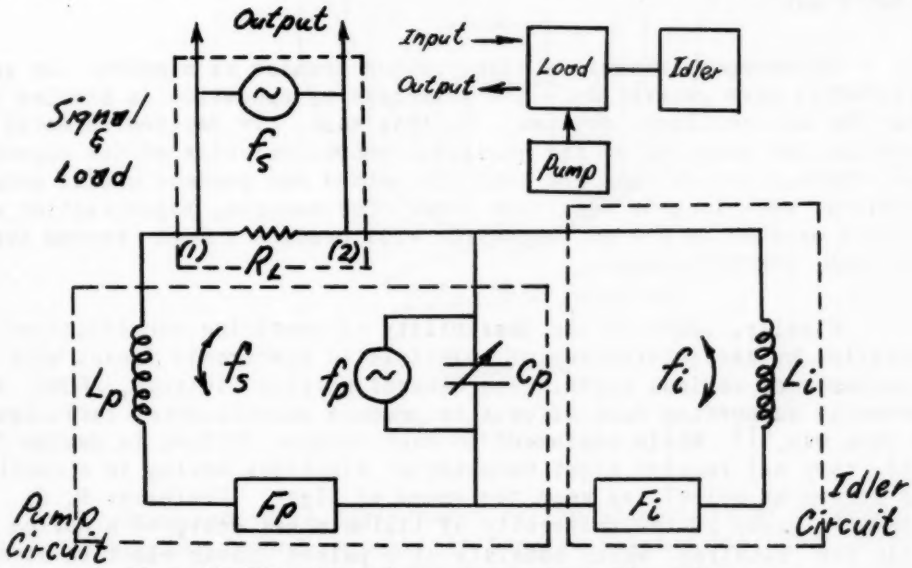


Fig. 3. Simplified schematic diagram of a parametric amplifier circuit. F_i and F_p are filters in the idler and pump circuits to isolate the idler and signal frequencies, f_i and f_s, from each other. L_i and L_p are idler and pump circuit inductances. R_L is the load resistance, f_p is the pump frequency and C_p is a capacitance that can be varied electronically.

The difficulty with this arrangement lies in the fact that the pump frequency must be slightly greater than the signal frequency due to inherent losses in the system. The pump frequency, however, is limited by the same considerations that apply to any high-frequency oscillator. In spite of this, however, parametric amplifiers operating at 100 gc have recently been designed, and indications are that they can be made to operate successfully at these and perhaps higher frequencies.

A number of possible submillimeter wave generators have been proposed that are based upon the nonlinear characteristics of certain key

circuit components. Where the capacitive or inductive element of an oscillator circuit incorporates materials having permittivities or permeabilities that vary nonlinearly with field strength, harmonics of the impressed frequency will be produced. This will also occur in variable resistance devices such as those produced by Professor Walter Gordy of Duke University. Professor Gordy has pioneered in the development of this type of oscillator under contracts and grants funded by the General Physics Division, AFOSR, since 1952.(6) Using this method, Gordy has successfully measured spectral resonances at wave lengths as short as 0.6 mm. His studies provided results that were published in 18 separate scientific papers last year. The practical power level of Gordy's technique, however, appears to be limited to a good bit less than 1 watt. This is because the power available decreases by about a factor of 10 for each subsequent harmonic and the resistive elements he uses burn out with input powers of about 1 watt.

Ferromagnetic devices also provide promise as a method for submillimeter wave generation. The principle of operation is similar to that for RLC nonlinear devices. In this case, the desired harmonic frequencies are produced by the nonlinear characteristics of the magnetization curve. Indications are that the method can produce useful energy levels at wave lengths less than 2 mm. For example, magnetization of a ferrite excited by a 4 mm magnetron will produce a 2-mm second harmonic and 1-mm fourth harmonic.

Finally, there is the possibility of producing submillimeter radiation by the interaction of relativistic electronic fields with structures of various types. The General Physics Division, AFOSR, is currently supporting four efforts to produce submillimeter radiation in this way.(7) While equipment in each program differs in design detail, they all require tight bunching of electrons moving in a continuous stream at velocities near the speed of light. Professor P. D. Coleman's group at the University of Illinois has designed a device he calls the "rebatron" which consists of a pulsed linear electron accelerator and a "bunching" cavity. Electrons moving near the speed of light from the accelerator are alternately delayed and speeded up in the cavity so as to produce high-velocity packets or bunches of electrons. These are further accelerated and passed into another cavity in which modes of the bunching frequency are produced. Submillimeter radiation is then extracted from this second cavity by a suitable wave guide. A 3-gc rebatron has been constructed and theoretical studies indicate that useful power can be obtained from harmonics above the 30th (about 100gc).

The relativistic electron submillimeter generator designed by Professor Eric Brannen, at the University of Western Ontario, differs from Professor Coleman's in the design of the accelerator. Professor Brannen calls his accelerator a "race-track microtron," a simplified

diagram of which is shown in Figure 4. This accelerator achieves higher velocities by recycling the beam through the accelerating potential. Improved bunching is also obtained as a result of the 4-sector design of the magnet.

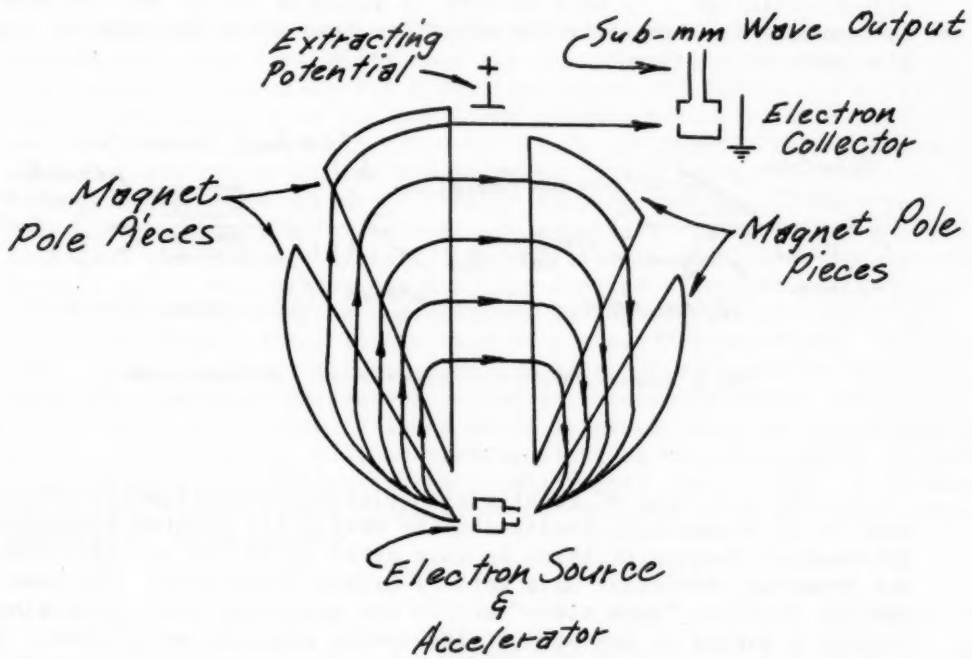


Fig. 4. A submillimeter wave generator using a "Race-Track Microtron" accelerator.

Professor R. A. Whitehurst at the University of Alabama, also under a grant funded through the General Physics Division, AFOSR, is studying a new approach to the production of submillimeter radiation by the Cerenkov effect. The Cerenkov effect, named after its Russian discoverer, was first observed as light produced in a dielectric material when relativistic electrons are decelerated in it. Professor Coleman also studied this technique under an earlier grant supported, in part, by AFOSR. His apparatus is diagrammed in Figure 5(a). A prototype of this device has been built using a 35-gc rebatron for the bunched-electron source. The ultimate hope is to obtain a frequency greater than 300 gc from the 9th harmonic at an output-power level of more than ½watt.

Professor Whitehurst's apparatus differs from Professor Coleman's in the use of a Stanford disc-loaded type linear accelerator as the relativistic electron-beam source, and in the use of a slow-wave struc-

ture instead of the teflon dielectric cone. This apparatus is diagrammed in Figure 5(b). The slow-wave structure is a corrugated metal cylinder, somewhat like those used in backward-wave oscillators, whose function is to provide fields with which the beam interacts to produce Cerenkov-effect radiation. In both devices, a suitable filter must be provided to separate the submillimeter radiation from other frequencies that are also present.

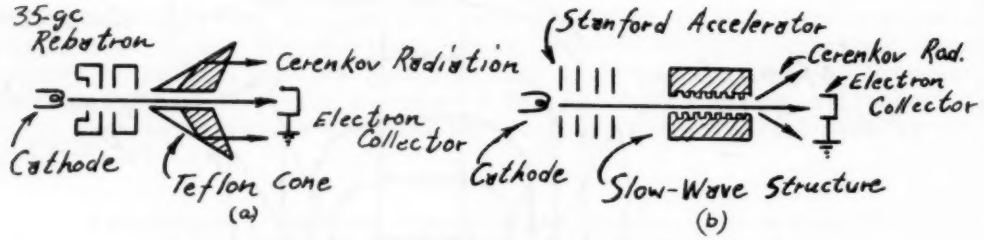


Fig. 5. Diagrams of two types of Cerenkov radiation submillimeter generators.

The last type of relativistic-electron submillimeter wave generator to be discussed (representing the only other program supported by the General Physics Division in this area) is called an "undulator" by its inventor, Professor Hans Motz of Oxford University. The name is derived from the "undulatory" motion the electrons take in passing through a series of magnetic fields having opposite polarities. This oscillating movement of the electron bunches generates an electric field perpendicular to the magnetic fields such that their combination produces an electromagnetic signal radiated towards the detector, as shown in Figure 6. The undulator has the advantage over both Cerenkov

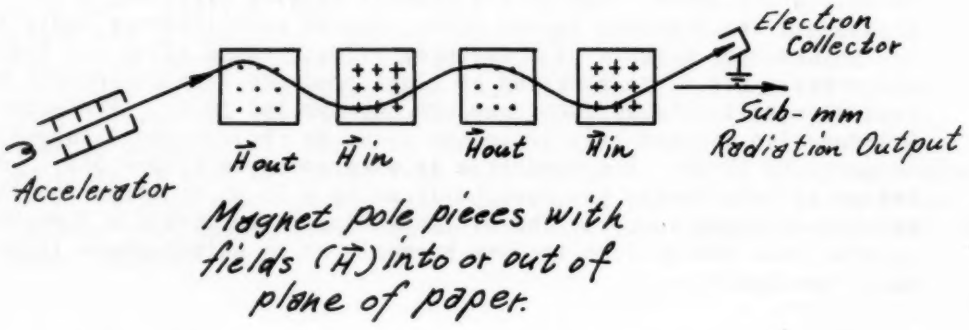


Fig. 6. Diagram of a relativistic-electron "Undulator" for submillimeter wave generation.

techniques in that the resulting radiation is propagated in free space without the need of intervening dielectrics or slow-wave structures. Initial experiments have produced about a milliwatt of power at wave lengths of about 1 mm.

Successful generation of submillimeter radiation by any of the methods discussed here would open the last remaining portion of the electromagnetic spectrum for scientific study and commercial exploitation. Some benefits to the Air Force which are likely to follow the successful development of such oscillators are greatly improved communications systems, including more compact transmitters, receivers and recorders with wide ranges of useable frequencies; vastly improved solid-state devices of all sorts; and sensitive night-and-bad-weather detection and guidance systems. In addition to such direct applications, submillimeter wave generators can also provide a nondestructive means of obtaining the electron temperatures in important high-temperature plasma research. Also, in the case of certain nuclear-fusion reactions, they can be used for the determination of reacting particle densities, which are important parameters in fusion-reaction studies. Finally, the study of high-velocity shock waves in gases, such as those produced by re-entry vehicles, will be greatly aided by the development of generators producing radiation in the submillimeter "gap." This is because such shock waves produce plasmas having characteristic or cutoff frequencies in this region.

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some aspects of modern MATHEMATICS

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1. What is Mathematics?

Everyone who has been exposed to the study of mathematics (say, on the college level) will recall encountering a number of different branches of this science. Some of these were algebra, geometry, and analysis. Analysis, for example, can be further subdivided into real analysis, complex analysis, functional analysis, and other areas. The various fields of mathematics deal with a variety of objects: some deal with numbers, some with geometrical figures, and others with completely different kinds of objects. The question naturally arises: What are the common properties of all mathematical fields which make us look at them as subfields of one and the same scholastic discipline, i.e., mathematics?

An attempt to answer this question completely would be the subject of a lengthy essay, which we do not intend to present here. We shall merely indicate one such common property, which is of fundamental importance.

Consider an area of human study and thought such as history or philosophy. Suppose you pick up a book or an article in one of those areas. Consider some of the fundamental expressions you encounter there. They may be words like "influence" (say, the influence of one philosopher on another), "reason" (e.g., the reasons for some historical event), "idea," "aim," "result," etc. Such expressions cannot be very sharply defined, and they may mean in a given instance somewhat different things to different people. Furthermore, in such a science, giving the "reasons," "aims," "influence," etc. of something or someone is often a matter which is highly subjective; we very frequently lack those objective criteria which can be used to check the validity of a given theory.

Consider, on the other hand, a book or an article in mathematics. Here the technical terms mean exactly the same to any two mathematicians. These terms are much more clearly and sharply defined than the basic concepts used, e.g., in history or philosophy. A statement like "$1549^{1843} + 7$ is prime," whether true or false, has some clear, unquestionable meaning which is absent in a declaration that "Shakespeare was the greatest playwright." The clarity and logical solidness of mathematical concepts and reasoning are apparent from the fact that, (apart from some questions concerning the foundations of mathematics, questions with which just very few mathematicians are concerned), there are no controversies or differences

of opinion in mathematics. More accurately, (with the exception just hinted at) a mathematical statement (called a "theorem") is considered (by all mathematicians) either to be proved or unproved. If the latter, then it is not considered in mathematical discourse as a true statement on which one can rely. Further, when some of the unproved theorems are proved to be false, their falsity is universally accepted.

Other unproved theorems (like Fermat's last theorem: if n is an integer larger than 2, there do not exist integers x, y, z, all different from zero, such that $x^n + y^n = z^n$) remain moot questions. They have not been proved; but neither has anyone proved their falsity. (Incidentally, one of the greatest achievements of modern mathematics was to show that there are "undecidable" theorems. These are theorems for which it is proved that they can neither be proved, nor disproved!)

One may say that an area of study can be considered as a mathematical field if, and only if, in addition to some requirements (which we shall not consider here), the concepts and the type of reasoning used therein are extremely clear and unquestionable. Thus, mathematics cannot include any area of study which depends, e.g., on one's emotions or artistic tastes. Such factors are too complicated and subjective to permit mathematical treatment.

2. Mathematical Rigor

We stated above that the mathematical fields are characterized by the extreme clarity of their concepts and reasoning. This, of course, has been apparent to mathematicians throughout the ages. However, one of the greatest achievements of 19th and 20th century mathematics was the remarkable strengthening of this clarity. A tremendous amount of work was done in these two centuries toward clarifying basic concepts (such as number, function, limit, area, set, etc.) and rewriting whole chapters of mathematics in a logically sounder fashion. Consequently, contemporary mathematics differs from that of older generations not merely because of its larger volume of knowledge; it is also much clearer (or, to use another term, much more rigorous) than the mathematics of the past. If one compares a modern textbook in mathematics (e.g., in calculus) with one on the same subject which appeared in the first half of the 19th century, he will be impressed to see how much clearer and more convincing the new presentation is, and how much vaguer the old textbook is, resorting to "handwaving" to settle difficult points (or, even worse, ignoring them altogether).

The fact that today mathematics is so crystal-clear in its reasoning does not imply that mathematicians do not use intuition, imagination, guesses, and heuristic reasoning in their work. Such processes are in fact very essential for the development of mathematical discoveries and advancements. However these processes, which every mathematician uses

as guidelines and motivations, are not by themselves considered legitimate proofs of theorems.

The high standard of rigor required nowadays from a mathematical work is not a matter of undue pedantry. There are good reasons for it. We shall not go into this matter in detail, but merely point out why we cannot accept experimental verification of a theorem as its proof in the same manner as we do in experimental science.

Frequently, a theorem deals with such a huge variety of different cases that it is clear from the outset that, by verifying the theorem for some (and even for many) individual cases, one gains no confidence that the theorem is true in general. For example, consider the following theorem proved by Lagrange (1736-1813): let n be a positive integer; then there exist integers a, b, c, d such that $n = a^2 + b^2 + c^2 + d^2$ (i.e., "every positive integer is the sum of four squares"). Suppose that some contemporary mathematician happens not to know about this theorem, but verifies it for many n's, say for n = 1, 2, 3, ..., 1000. Although he may be inclined to feel that the theorem holds in general, his success in proving it for n = 1, 2, ..., 1000 is not considered a proof of the general theorem, nor will he himself be convinced of its truth. In fact, among the integers larger than 1000, there are so many which differ from all the numbers 1, 2,. . . ., 1000, in so many respects, that the fact that the theorem was verified for n = 1, 2, 3,...., 1000 cannot make one feel sure of its general validity.

The following fact should be of interest here. The mathematician, Euler, (1707-1783) found by checking all odd numbers larger than 1 and smaller than 2500, that every such number can be represented in the form $p + 2m^2$ where p is prime and m is an integer. This caused the mathematicians of that time to believe that every odd number larger than 1 had such a representation. However, later mathematicians checked all odd numbers larger than 2500 and smaller than 9000, and found among them exactly two, namely 5777 and 5993, which were not representable in the above form.

3. An Expanding Science

We have emphasized above how modern mathematics distinguishes itself from older mathematics by being more rigorous. This, however, should not conceal the fact that modern mathematics is characterized also by the tremendous and permanent expansion of its branches, by the constant creation of new branches, and by being able more and more to be applied in other areas of human activity.

Mathematicians are constantly solving important problems, some of which have defied all attempts at solution for many years. Some of these solutions, whether full or partial, are achieved by developing new methods which are frequently very valuable apart from their appli-

cation to these particular problems. In fact, sometimes such a method, by itself, is by far more important than the solution of the problem which gave rise to it.

The expansion of the various mathematical branches and the creation of new ones have been so intensive that today, if two mathematicians meet by accident, the chances are that one knows just a little (if anything at all) about the area of work of the other. It is interesting, however, that a reverse trend is also taking place. New researches often point out a common feature of different mathematical branches. Sometimes a new theory is established which unifies a number of previously unrelated theories into a new, compact branch of mathematics.

There is today an ever-increasing use of mathematics in other sciences. These applications are not limited merely to physics, engineering, chemistry, astronomy and related areas, but can also be found in biology, economics, and in the operation design of military, industrial and business organizations. In more and more areas of human activity, the services of the mathematician are being requested. And he may offer his help in any situation where the problems involved can be so formulated as to be manageable by the kind of reasoning used in mathematics.

AFCRL MOSSBAUER APPARATUS

LA VERNE E. WOODS, *Research Information Office*, AFCRL

AFCRL has designed and is building a high-pressure Mossbauer apparatus for analyzing the magnetic properties of crystals. Mossbauer techniques promise to give a completely new insight into the structure of materials, and have been employed in recent months by several organizations for analyzing crystals. The AFCRL-designed equipment is marked by the high precision of its moving components. The Mossbauer effect is concerned with the induction of resonance in a material. This resonance results when a radioactive source, such as cobalt 57, emits gamma rays which are absorbed and re-emitted by a second material. Because of the very short wave lengths involved, and because of Doppler considerations (or, from another framework, considerations of quantum mechanics), the source of gamma radiation must be moved very, very slowly with respect to the absorbing material if the material is to resonate. The precision engineering necessary to compensate for very small frequency shifts distinguishes the AFCRL Mossbauer apparatus from others. (SOLID STATE SCIENCES LABORATORY)

THE MYSTERIOUS PHENOMENON OF *retrograde* MOTION

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High-heat transfer from an electric arc to the electrodes limits the performance of today's high-power plasma generators. Moving the arc rapidly along the electrode surface through the use of an external transverse magnetic field considerably reduces this thermal load. Ampere's law determines the direction of this induced arc motion. However, under certain circumstances, the motion of the arc may become stationary or reverse its direction, seemingly violating this well-established law.

This phenomenon of retrograde motion, i.e., the apparent motion of the arc discharge against the Lorentz force, has for quite some time been of considerable research interest, but still appears to be one of the mysteries of the electric arc. Numerous investigators in the past, through 30 or more papers, have tried to investigate this phenomenon, but it is still unsolved. Most studies have been concerned with low-current discharges emphasizing the cathode region. No consideration has been given to the associated column.

At the Thermomechanics Research Laboratory, an experimental study on this phenomenon has resulted in some new observations which were obtained under well-defined and controlled experimental conditions. The apparatus was specifically designed to permit high-speed photographic studies of the entire discharge at higher currents than previously reported. It consisted essentially of two concentric, water-cooled copper-ring electrodes enclosed in a low-pressure chamber covered by a quartz window and surrounded by an air-core solenoid magnet. The chamber was filled with argon kept at constant temperature and controlled pressure. The direction of the gas motion near the electrodes induced by the discharge was indicated by a small torque meter. The arc velocity was determined by photoelectric counting of the arc rotations, averaged over many cycles. The variable test parameters covered the following ranges:

Gas Pressure	-	4 - 500 mm Hg.
Magnetic-Field Strength	-	500 - 4,000 gauss
Arc Current	-	50 - 200 amperes

Figure 1 shows some of the experimental data indicating lines of constant retrograde velocity as functions of pressure and magnetic-field strength. The line of zero velocity indicates the critical conditions of pressure and magnetic-field strength under which the arc

velocity has decelerated to zero. Under the existing experimental conditions, zero velocity could be obtained at pressures up to 500 mm Hg. At higher pressures, the arc became unstable and reproducible measurements could not be made. It is felt that retrograde motion may be obtained at atmospheric and higher pressures.

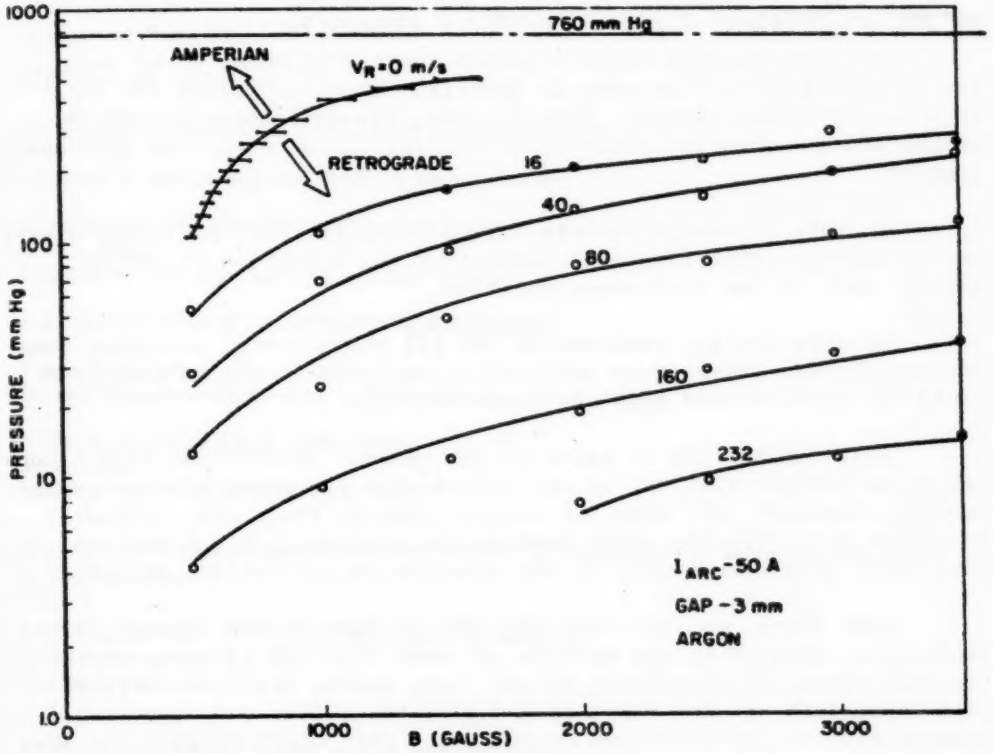

Figure 1 RETROGRADE VELOCITY

As a most interesting observation, it was found that the arc induces, as a pump, a flow of neutral gases always in the correct or Amperian direction, independent of the motion of the arc discharge itself, i.e., whether the arc moved forward, remained stationary, or moved in retrograde direction.

In summarizing some of the experimental findings, the following may be stated:

1. In contrast with the common belief, retrograde motion is not restricted to the cathode region of the discharge. For short arcs between parallel electrodes, the entire discharge moves along the electrodes.

2. The speed and direction of the motion of the discharge is dominated by certain combinations of pressure, current and field strength.

3. The phenomenon of retrograde motion is not necessarily restricted to the low-pressure regime. Zero velocity and retrograde motion are obtained at near-atmospheric pressures.

4. Interruption of the cathode surface by means of an insulating strip delays the arc when in Amperian motion and stops the arc when in retrograde motion. When stopped, the deflection of the arc column changes from the Amperian to the retrograde direction with anode lagging.

5. The discharge induces a flow of neutral gas particles always in the Amperian direction, independent of the direction of the arc motion, even in the stationary position.

No quantitative explanation for all the observed phenomena can yet be offered. However, a qualitative explanation may be suggested along the lines of one given by Stark in 1903.

This explanation is based on the general observation that a concentrated cathode spot may be the origin of a streaming of electrons and ions (cathode jet) from the cathode towards the anode. (Usually ions move only from the anode towards the cathode.) Stark believed that these jets were caused by the vaporization of cathode material.

Like Stark, we feel that the flow of ions in the "wrong" direction, i.e., carried by the cathode jet away from the cathode, may be the main cause for retrograde motion; but, unlike Stark, we believe that these jets are not vaporization jets but rather the magnetically induced cathode jet described by Maecker. This could explain the influence of the gas pressure on the motion of the column as an effect of the penetration depth and particle velocity of this cathode jet, since both will increase with decreasing pressure.

In addition to the effect caused by the cathode jet, similar arc pumping through interaction of the external field with the discharge may also be responsible for the retrograde motion of the arc. This pumping effect is asymmetrical with respect to the axis of the arc. It will induce a gas flow normal to the arc's axis moving from the retrograde side to the Amperian side, thus causing a pressure difference at both sides of the column. Interaction of the superimposed effects of these two magnetic fields will determine the final motion of the arc as functions of the prime parameters. Calculations are being planned to check the validity of this model.

This information was reported at the Sixth International Conference on Ionization in Gases at Paris, France, July 1963.

to our readers

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OAR RESEARCH REVIEW is published biweekly by the Office of Scientific and Technical Information (RRYC), Office of Aerospace Research, USAF, Washington, D.C. 20333. The objective of this publication is to make a positive contribution to the exchange of information concerning Air Force sponsored and conducted basic-research activities. Individuals or groups associated with basic research may obtain copies by written request to the above address.

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